10-14

BELLCOMM, INC.

SUBJECT: Improvement to the Secant Method - Case 103-2

DATE: September 27, 1966

FROM: J. J. Schoch
R. T. Yuill

ABSTRACT

An improvement to the n-dimensional secant method is presented. The secant method is employed in subroutine Wolfe for the solution of simultaneous, not necessarily linear equations. It is iterative and at the end of each iteration the new solution is used to replace the old solution having the largest error. Occasional difficulties have been experienced with non-convergence.

The subject improvement changes the criterion for selecting the solution to be replaced by specifying that each new solution must be used at least twice before it may be replaced. It has yielded convergence in cases that did not converge using the existing method.

Graph 1 shows how a case that did not converge properly when using the old method, converges normally when using the new criterion.

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MEMORANDUM FOR FILE

Introduction

In using the secant method for trajectory targeting, cases were encountered where no convergence was obtained.

A closer investigation showed that changing the criterion by which one of the old solutions is eliminated, convergence is obtained.

The Present Secant Method

A more complete explanation of the secant method may be found in (1) and (2) while its application to targeting in launch vehicle applications was mentioned in (3).

For the present discussion, it will be sufficient to remember that the secant method determines the values of n independent variables in order to reduce the error of n dependent variables to an arbitrarily small value by an iteration process. During each iteration, a better value of the independent variables is obtained by appropriately manipulating the errors of some of the previous approximations. As a new approximation is obtained, one of the old ones is eliminated.

Wolfe (1) suggests elimination of that solution for which the sum of the squares of the errors in all dimensions is maximum but indicates that there might be other criteria that are more effective. The present version of the secant method (4) computes a certain weighted sum of squares and eliminates the solution having the largest sum.

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(1) Numbers apply to Reference at the end.

[redacted]

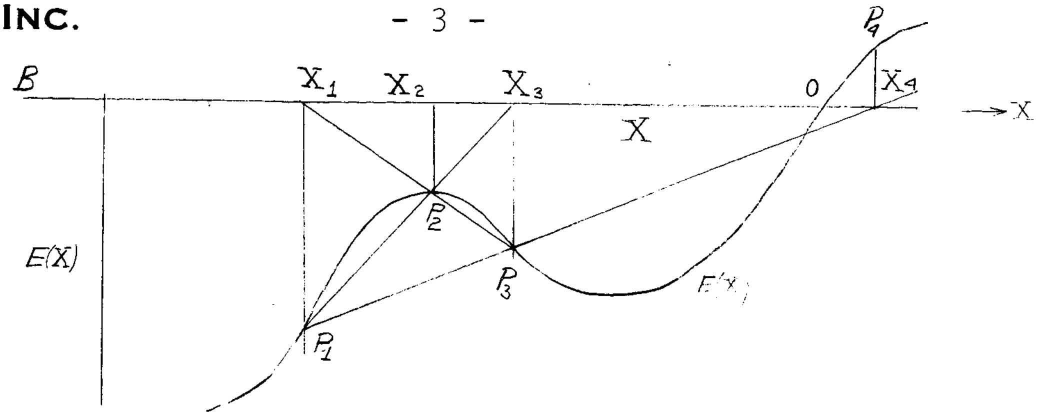
In a typical application of the secant method for targeting in an earth orbital trajectory, an attempt is made to adjust for instance, payload and certain guidance parameters to obtain a horizontal flight path angle and proper velocity and altitude. Such a typical run may have an initial altitude error of between 10^5 and 10^6 feet and converge to an allowable error of ten feet within six to nine iterations, which means that the error decreases on the average by a factor ten for each iteration.

Graph 1 shows a plot of altitude error versus iteration for a non-convergent case. The curve labeled "Present secant method" shows how the present criterion of selecting the approximation to be eliminated does not lead to convergence. The altitude error starts for the first iteration at about 340,000 feet and ends with an error of about 150,000 feet. The corresponding plots for velocity and flight path angle error show a similar trend.

Reason for Not Converging

Closer investigation of the case showed the following. The errors of three dependent variables are reduced by changing the value of three independent variables. At any time, four approximations are used simultaneously to find a better one. The approximation having the largest error is replaced by the new approximation. Since the new approximation again has the largest error, it will be eliminated this time also and the same three old solutions kept, and so forth. This simply means that the secant method operates over and over with the same old three points combined with a fourth one which keeps changing a little bit.

The situation, though impossible to visualize in four dimensional space, may be encountered when the hyper-surfaces have a local extreme point. For clearer understanding, reference is made to the figure on the next page showing a one dimensional case having a local extreme point.



Let E (x) be the error function which should be reduced to an arbitrarily small value. The desired solution is in the vicinity of point 0. It will be shown graphically that if the iteration process is started in the vicinity of point P_2, it cannot converge to 0.

Let P_1 and P_2 be two initial solutions. Drawing the secant through them leads to point X_3 with corresponding P_3. Now the point with the largest previous error, P_1 is eliminated, and the new point P_3 is used together with the remaining one to draw the new secant which on the figure leads back to point X_1. This situation may look unique but is in reality a limiting case which would eventually be reached after a large number of iterations. The result is that with this elimination criterion, there is no chance of ever reaching point 0 because point P_2, having the smallest local error, can never be eliminated and the iteration process is locked in an area surrounding the local maximum.

The Improved Secant Method

By tabulating the number of times the same approximation may be kept and limiting that number, there would be at least a chance of converging to the right point. If for instance after having used point P_2 several times it is decided not to use it again but to use the other two points P_1 and P_3 instead, a new intersection at X_4 (dotted line) is found providing a point P_4 and the method will eventually converge to point 0.

Should, due to a different relative position of points P_1 and P_2, the intersection between these points and the E = 0 line have fallen on the opposite side of the local maximum, for instance to point B, then convergence would not be assured. However, it still would have more of a chance to converge than if it remained locked around the local maximum. This is the same approach that was applied to the n-dimensional case. The improved method specifies that no new solution may be eliminated unless it has been used for determining at least two approximations.

This technique was applied to the problem that showed no convergence on Graph 1. The result is shown on the same graph on the curve labeled "Improved secant method." The problem converges this time in nine iterations.

Conclusions and Recommendations

/ In order to avoid the difficulties encountered by the writers it is recommended that users of the secant method incorporate the proposed changes in their subroutine. The improved secant method called subroutine, Wolfe-1 is available from the Computer Library.

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Attachment:
Figure 1
References

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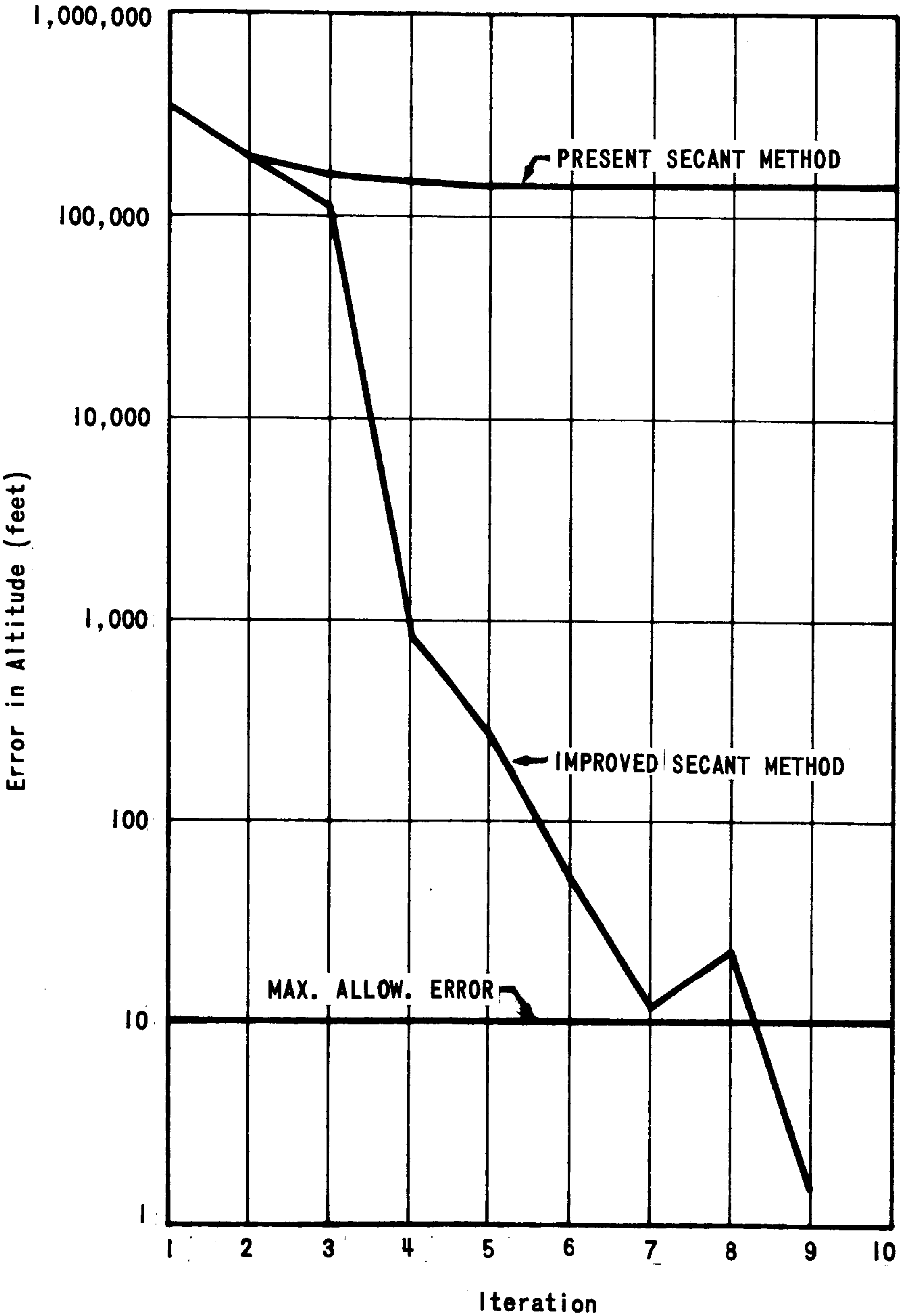
(2) "An Explanation of the Secant Method as Proposed by Wolfe for Solving a System of Simultaneous Non-linear Equations" Case 103-4, J. J. Schoch, Bellcomm, Memorandum for File, March 1, 1966.

(3) "Trajectory and Targeting Computer Program," J. J. Schoch, Bellcomm Technical Memorandum, TM-66-1021-2, March 1, 1966.

(4) "Subroutine Wolfe," W. B. Gragg and J. M. Ortega, available at the Bellcomm Computer Library, August, 1964.

ALTITUDE ERROR VS. ITERATION
FOR THREE DIMENSIONAL CASE



GRAPH I